Math 10 Highlights

Chapter 1: Systems of Linear Equations and Matrices

- Linear Systems
  - Gaussian/Gauss-Jordan
  - Solution possibilities
- Matrix Computation, Using and Proving Properties
  - Add/Subtract/Scalar Multiply/Multiply
  - Inverse
  - Transpose
  - Elementary
  - Zero
  - Identity
  - Diagonal
  - Triangular
  - Symmetric
  - Skew Symmetric
- Big Theorem

Chapter 2: Determinants

- Calculating determinants by Cofactor Expansion
- Properties of determinants
- Affect of row operation on determinants
- Computing determinants using row reduction
- Cramer's Rule
- Relationship between determinant and invertibility…Big Theorem

Chapter 3: Vectors in $R^n$ .

- Computation, Length, Distance, Angle
- Properties, use and prove

Chapter 4: Vector Spaces/Bases

- Vector Space
  - Vector Space definition/ Ten Axioms
  - Prove/Disprove given space is a vector space
  - Properties in Thm 4.1.1
  - Common Vector Spaces
- Subspace
  - Definition: Nonempty plus two conditions
  - Prove/Disprove given space is a subspace
  - Linear Combination
  - Span: def/ form subspace?/ given set of vector span given space?
- Linear Independence
  - Definition
  - Properties
  - Determine whether a give set of vectors is linearly independent.

Determine whether a give set of functions is linearly independent.
Properties

Basis and Dimension
- How do we show a set is a basis?
- Express a vector in terms of a given basis (coordinates)
- Standard bases for common vector spaces
- Dimension
- Properties relating dimension to independence, span, basis.
- Enlarging a linearly independent set to form a basis (thm 4.5.5)
- Reducing a spanning set to form a basis. (thm 4.5.5)

Row Space, Column Space, Null space.
- Find bases for each
- Find rank, nullity
- Relationship to solutions of system $A\vec{x} = \vec{b}$
- Results in Big Theorem.

Change of Basis
- Find the transition matrix from one basis to another.
- Convert $[\vec{v}]_B \rightarrow [\vec{v}]_{B'}$

Linear (Matrix) Transformations $\underline{R^n \text{ to } R^m}$
- Geometric Operators (Reflection, Rotation, Projection)
- Composition of transformations
- One to one transformation
- Inverse transformation
- Properties relative to Big Theorem
- Use Thm 4.10.2 to prove/disprove transformation is Linear.
- Find standard matrix A such that $T(\vec{x}) = A\vec{x}$ directly or using

$$A = \left[ T(\vec{e}_1) \middle| T(\vec{e}_2) \middle| \cdots \middle| T(\vec{e}_n) \right]$$

- If T is a linear transformation then $T(\mathbf{0}) = \mathbf{0}$

<u>Chapter 5: Eigenvalues, Eigenvectors</u>
- Eigenvalues
- Geometric Interpretation
- Eigenvectors
- Eigenspaces
- Diagonalization

<u>Chapter 6: Inner Product Spaces</u>

Inner Products
- Definition – 4 axioms
- Prove/Disprove a given operation is an inner product
- Computation
- Properties
- Length/Distance Props. including Cauchy-Swarz, Gen. Pythagorean
- Orthogonality

Orthogonal Complement

- Properties
- Row space orthogonal complement of null space

Orthogonal/Orthonormal Basis

- Orthogonal/orthonormal set
- Computing coordinates relative to Orthogonal/Orthonormal Basis
- Properties
- Computing Projections
- Converting a given basis into an orthonormal basis (Gram-Schmidt)

Chapter 8: General Linear Transformations

General Linear Transformation

- Prove/disprove transformation is Linear.
- Find the matrix for T with respect to the bases B and B' and use it to compute $T(\vec{x})$